[redacted]

5-Person Team Test

Abbreviated Instructions: Answer each of the following questions **using separate sheet(s) of paper for each numbered problem**.

- Place your team code in the upper right corner of each page that will be turned in.
- Place problem numbers in the upper left corner (failure to do these things will result in no score for that problem/page).

Problems are equally weighted; **teams must show com lete solutions not 'ust answers to receive credit**. More specific instructions are read verbally at the beginning of the test.

1. Given a square of unit area:

   a. Show that the square can be partitioned into six squares (Note: The squares do NOT need to be congruent.)

[redacted]

4. A circle has both an inscribed and circumscribed regular polygon (both having the same number of sides). Find the ratio of areas for the larger polygon to the smaller.

   a. If the polygon is a triangle.
   b. If the polygon is a square.
   c. If the polygon is a hexagon.
   d. If the polygon is has n sides. As n gets large, what number does the ratio approach?

5. Jayden and Cody decide to play a coin flipping game. They decide to flip a fair coin until they obtain a sequence of either five consecutive heads or five consecutive tails, at which point the game will end.

   a. What is the probability the game ends within the first five flips?
   b. What is the probability the game ends within the first six flips?
   c. What is the probability the game ends within the first seven flips?

Problem #2

2 Le be the proc le nts
and s be the sum of elts in P. Then since
he sum of all ten numbers $\frac{10(11)}{2} = 55$, we
have the $p = 55-$ n $s = 5$ for
numbers t o sum oc

Note t $.2.3.4.$ > 5 so o product
h satisfies le e c four will have
more than umbers. lso there canno be
only one number, h node $x = 55$ has
so ba fo ≤ 1.

(The solutions are $P = \{6,7\}$, $P = \{1,4,10\}$ and $P = \{1,2,3,7\}$

2 e l : Tr or cases Cal second lement

$x = 1$. +) = ... $\Rightarrow$ $2y = 54$ no Solution
$= 2$ 2 $+(2+y) = 55$ $\Rightarrow$ $3y = 53$ " "
etc $\Rightarrow$ to $x = 5$ no sol'n.
$x =$ 6 $+(6+\ )=55$ $\Rightarrow$ $7\ = 49$ $\Rightarrow 7$
$=$ 7 $(7+$ $= 55$ $8\ = 48$ $\Rightarrow 6$ repeat!)
$x = 8$ $8y$ + + $= 55$ $= 4$ no sol'n
$x = 9$ 9 ($) = 55$ $10\ = 46$ "
$x = 10$ 10 $(10+$ $= 5$ $11\ = 4$ "

(4) 2 o t s' l (

5 e ne x $+(x+y$ $\epsilon = 55$

not all odd is also odd.
ot 4 odds ca sum to 55.

In one odd sq is o l

ven n 5 xyz + (x+ z) = 55 is on)

possible is u on_y

one odd

xy 2 even.

2·4·z + (2+4+ = 5. 9z=49 No Sol

1 2 =55 3 =4 " "

1 + z+0+ = 17 =45 " "

2·10 z + z+10+ = 55 21z=43 " "

·6 + (+6+ =55 25z=45 " "

4·8 z + (4+8+) =55 33z=43 " "

* ·10 +10+z)=55 41z=41 z=1

[illegible]

·10 + z= 61z= " "

or 3 u (4 0)

Heck = (x, x. deg

1·2·4·5 + (1+2+4+ = 54 2 + 5 No

1·2·4 + +3+5) = 55 30+11 55 No

·2·3· + (1+2+3+6) = 55 No

1·2·3·5 + (1+2+3+5) = 42 + 13 = 55 Yes

1·2·3·x Answer x > ·11 No

its end 1,2,3

Problem #3

3) (1, 2, 3, 4 +, - * aren[d]theses

a) Max: $19 = 4*(3+2)-1$

b) Min: $-19 = 1-4*(3+2$

c) 5_5 : $+2-3*4=0$

Problem #4

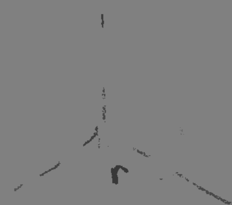
$x \quad 2x$

$x \quad r\sqrt{3}$

SmallΔ $A = 6 \quad b \quad = 6 \; 1/2 \left(\frac{3r}{2}\right. \quad \overline{2}$

$=$

$A = 6 \; \frac{1}{2} b h = 6\left(\frac{1}{2}\sqrt{3}r\right) r) \quad = 3\sqrt{3} r$



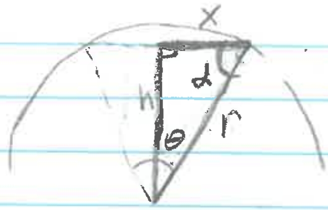
$3r \quad 30$

$r \quad 60$

$r \quad$ larc $\quad - \; 3 \; 3 \; ^2 \quad -$

$r \quad 2$

general



Small n-gon $A = \frac{1}{\ } x \cdot \ = \frac{1}{2}\ r \sin\ )\ r \cos\alpha = \frac{\ }{\ } r^2 \sin\alpha\cos\alpha$

$\sin\alpha = \frac{h}{r}$ $\cos\ = \frac{x}{r}$

α $= r \cdot \cos\alpha$

x

Lar n- A $x \cdot r = \frac{1}{\ } \frac{r}{\ }\ r$

$=$ $= \ ^2$

x

$x = $

a

2

$=$

Sm $2\tan\alpha = 2\sin\alpha\ \ \sin\alpha\cos\alpha$

$= \frac{1}{\sin}$

Problem # 5

5 5 Heads on a ls in a now

a 5H v 5 + of 5 rolls

$\frac{}{2} + \frac{}{2} = \frac{1}{2}) = 1$

or

$1 \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{} = 1/$

any same same same same

b) Prbl oct of = Pn (1st 5 st 5

$\cdot \left(\frac{1}{2}\right)^6 = \frac{1}{16} + \frac{1}{} = \frac{3}{32}$

H TTTTT
or
HHHHH

from

$3/ + 4 \cdot 1$

HT 5H
HH 5T
TH 5T
TT 5T

d) If we re eat part th nob likes of 1/3 a 2/3, we f

$\frac{}{3}$ 5 2 5 = $\frac{1 + 32}{243}$ $= \frac{33}{243} > 16$

So m r likely end e in

Problem #6

) $y^2 = x^2 + b$ c be wn #'s

$-{}^2 \quad y^2 - {}^2 = y - x) \quad + \quad =$

a =24 $(y-x)(y+ \quad = 2$

-actor pairs for 24: (1·24 (2·12 3·) (4·6

2,12 = = The c be no

(4·6 =5 =1 integ n 1 facts.

b) $b=60$ $(y-x)(+x = 60$

·6 2·30 / 3·20 / 4·15 / 5·12 6·1

2·30 =16 x=1 diffcne for

6·10 = x= the ch factor s.

c =210 He 21 = 2·3·5·7

no matter w you factor, in th e

1 be ven & one wil odd a so, as

above, there are no solutions