2021 John O'Bryan Mathematics Competition
5-Person Team Test

Abbreviated Instructions: Answer each of the following questions **using separate sheet(s) of paper.** Questions will not be scored without the following two things:

- Place your team code in the upper right corner of each page that will be turned in

$k =$

$+$

5. The integers from 1 to n are written in a row. The same numbers are written under them in a different order (see below). Note that there are no repeated numbers in a row.

1	2	3	4	5
4	2	1	5	3

[redacted]

=

6. The side BC of an equilateral triangle ABC is divided into three equal parts by the points K and L with K closer to B. The point M divides the side AC in the ratio [redacted]

=

$AM : MC = 1 : 2.$ + =

Freshman-Sophomore Individual Test

Directions: Please answer all questions on the answer sheet provided. All

[redacted]

9. The rational expression $\dfrac{\left(3-\dfrac{2}{1-x}\right)}{\left(\dfrac{3}{x-1}-1\right)}=\dfrac{kx+w}{p+qx}$ where k, w, p, and q are integers with $k>0$. Determine the sum $(k+w+p+q)$.

10. $\overline{AC}$ is the diameter of Circle O. Point B lies on the circle such that $\angle BAC = 60°$ and $AB = 10$. Determine the exact length of the radius of Circle O.

11. An array of 12 points is set up in 3 evenly spaced rows and 4 evenly spaced columns

2021 John O'Bryan Mathematical Competition
Freshman/Sophomore Individual Test

Note: **All answers must be written legibly in the correct blanks on the answer sheet and in simplest form. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required. Each problem has the same point value.**

1.

11.

3.

13.

4.

14.

5.

15.

6.

16.

7.

17.

8.

18.

9.

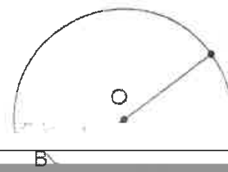
19.

10.

20.

2021 John O'Bryan Mathematical Competition
Junior-Senior Individual Test

Directions: Please answer all questions on the answer sheet provided. All answers must be written legibly and

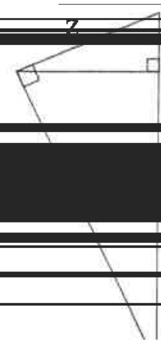


in simplest form. Exact answers are to be given unless otherwise specified in the question. No units of

13. Let $f(x) = \sqrt{a}\,x^2 + \sqrt{ab}\,x + b\sqrt{a}$ with $a > 0$ and $b > 0$. Determine the minimum value for $f(x)$ in terms of a and b

14. Trailing zeros are the zeros to the right of the last non-zero digit in the integer representation of a number read from left to right. How many trailing zeros does the number $n = 2021!$ have?

15. The triangle shown on the right, $(x + y + z) = 2\left[\sqrt{k} + \sqrt{w} + \sqrt{p}\right]$ in simplified and reduced radical form with k, w, and p positive integers. Determine the sum $(k + w + p)$.



16. An array of 12 points is set up in 3 evenly spaced rows and 4 evenly spaced columns

(forming 6 adjacent congruent squares if connected horizontally and vertically). One point is chosen at random from each row. Determine the probability the 3 points chosen

2

B(26,0)

Name: ______________________ Team Code: ______

2021 John O'Bryan Mathematical Competition
Junior/Senior Individual Test

Note: All answers must be written legibly in the correct blanks on the answer sheet and in simplest form. Exact answers are to be given unless otherwise specified in the question. No units of measurement

1. ____________	11. ____________
2. ____________	12. ____________
3. ____________	13. ____________
4.	14.
5.	15.
6.	16.
7.	17.
8.	18.
9.	19.
10.	20.

2021 John O'Bryan Mathematical Competition
Questions for the Two-Person Speed Event

*****Calculators ma not be used on the first four uestions*****

1. Let $\begin{vmatrix} 7 & 0 & 4 \\ x & 5 & -2 \\ -1 & 3 & 0 \end{vmatrix} = 158$ and $y = \dfrac{\log_4(27)}{\log_2(9)}$ Find the product xy.

2. Let 36, k, and 20 in that order be the terms in an arithmetic sequence. Let 12, w, and 48 in that order be terms in a geometric sequence with $w > 0$. Then the terms k, w, and p, in that order [redacted]

3. The point P is located on the x-axis and the point Q is located on the y-axis. Each point is 5 units [redacted]

Names: **School:**

2021 John O'Bryan Mathematical Competition
Answers for the Two-Person Speed Event

Note: All answers must be written legibly and in simplest form. Exact answers are to be given unless [illegible]

1.

2.

3.

4.

[illegible]

6.

7.

8.

SCORE

T1.

T2.

2021 John O'Bryan Mathematics Competition
5-Person Team Test

1. (a) Find prime numbers $p < q < r < s$ satisfying $p^q \cdot r \cdot s = 12008$ and $q \cdot r^p = 1083$

Solution: Since 12008 is even, 2 is a factor; factoring 2s gives $12008 = 2^3 \cdot 1501$.

which means $p = 2$ and $q = 3$. Since the sum of digits of 1083 is divisible by 3,

[redacted]

$361 \quad r^2$ implies $r = 19$. Hence $s = \frac{1501}{19} = 79$. Therefore $p \quad 2$, $q \quad 3$, $r \quad 19$, and $s = 79$ giving $12008 = 2^3 \cdot 19 \cdot 79$ and $1083 = 3 \cdot 19^2$.

2. A *ribbon of length* k is a rectangle with dimensions $1 \times k$ for some natural number k.

[redacted]

to cover the area has a different length. What is the maximum area that can be

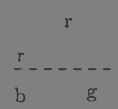
[redacted]

3. (a) For real numbers x and y, show that $|x + y| \le |x| + |y|$. (Hint: if $a, b \ge 0$ with

[redacted]

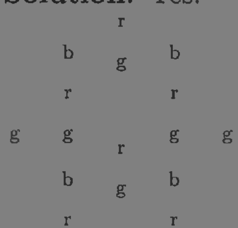
$\le |x+y-z|+|x-y+z|+|-x+y+z|$

[redacted]



(b) Can a hexagonal prism be colored so that every face and vertex see all 3 colors?

Solution: Yes.



(c) Can a rectangular prism be colored so that every face and vertex see all 3 colors?

[redacted]

[redacted]

1	2	3	4	5
4	2	1	5	3

Notice that the sum of numbers in the middle three columns is a perfect square. Is it [redacted]

(a) $n = 9$?

Solution: Yes. The sum of the number 9 and the number beneath it lies between 10 and 18. Since 16 is the only perfect square in this interval, the number under 9 must be 7. Similarly, the number 7 must be written above the [redacted]

6. The side BC of an equilateral triangle ABC is divided into three equal parts by the points K and L with K closer to B. The point M divides the side AC in the ratio

[redacted]

Name: ANSWERS Team Code:

2021 John O'Bryan Mathematical Competition

3. 4.5 Must be this decimal.

60

(10,17) Must be this ordered pair

2/15 Must be this reduced fraction.

$\frac{2636}{495}$ Must be this improper fraction.

8. 276

13.

32

1 and -1/3 Must have both answers in either order

Must be this

$2\sqrt{13}$ Must be this radical expression

Name: ANSWERS **Team Code:**

2021 John O'Bryan Mathematical Competition

1. 2

2. $2\sqrt{13}$ — Must be this radical expression.

3. 96

4. 2

5. -6

6. 5/6 — Must be this reduced fraction.

7. 5/3 — Must be this [illegible]

8. $-1/2$ — Must be this reduced fraction.

9. 137

10. 2

11. $-24/25$ — Must be this reduced fraction.

12. 325

13. $\frac{3}{4}b\sqrt{a}$ — Must be this exact answer.

14. 503

15. 41

16. 1/8 — Must be this reduced fraction.

[illegible] 406

18. 1/5 — Must be this reduced fraction.

19. 88.47 — Must be this exact decimal.

20. $\left(\frac{739}{[illegible]}, \frac{110}{[illegible]}\right)$ Must [illegible]

Names: ______________________________ School: ______________________

2021 John O'Bryan Mathematical Competition
Answers for the Two-Person Speed Event

Note: All answers must be written legibly and in simplest form. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required. Each problem has the same point-value.

SCORE

Calculators are not allowed to be used on the first four questions!

This competition consists of eight competitive rounds. Correct answers will receive the following scores:

1st: 7 points
2nd: 5 points
All Others: 3 points

There is a three minute time limit on each round. You may submit only one answer each round. To submit your answer, fold this sheet **lengthwise** and hold it high in the air so that a proctor may check your answer.

1. 6

2. 20

3. 10

4. $6+3\sqrt{2}+12\sqrt{3}$ Terms may be in any order

5. 56

6. 17

7. 256

8. 14.4 Must be exactly this decimal.

SCORE

T1. 0

T2. 50